# Implementation of path following techniques into the finite element code LAGAMINE. Part II: Material non linearity

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# IMPLEMENTATION OF PATH FOLLOWING TECHNIQUES INTO THE FINITE ELEMENT CODE LAGAMINE PART II: MATERIAL NON LINEARITY

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Abstract. This short report concerns the second part of the work performed in ArGEnCo in order to implement advanced incremental-iterative solution techniques into the finite element code LAGAMINE for geometrical and physical non-linear problems. More specifically, it deals with the implementation of a convergence criterion suitable for material non linearity and the validation of the approach on a Brazilian test on a concrete specimen.

## 1. Introduction

During the previous visit of the first of the authors in the ArGEnCo departement (Mars and May 05), a detailed report was written on existing advanced incremental iterative solution techniques for geometrically and physically non-linear problems, [10]. After a theoretical presentation of different path following techniques, a general algorithm and details about the specific implementation into the finite element code LAGAMINE were given. Challenging examples, mainly dealing with geometrically non linear phenomena, were used to validate the approach.

This second part of the report deals with the implementation of a convergence criterion suitable for physical (material) non linearity. The performance of the approach is validated using a Brazilian test on a concrete specimen.

## 2. Convergence criterion

For geometrically non linear problems, the determinant criterion was chosen in our first report, [10]. More specifically, one has to store the sign of the determinant of the stiffness matrix at the end of each step and to look the existence of a negative pivot in the triangularization of the global stiffness matrix at the beginning of the new increment. This is an indicator that the determinant of the tangent stiffness matrix changes sign and that a critical point has been overcome.

However, the determinant criterion was not found to work properly for material non linear problems. As mentioned in [8], this criterion is not adequate when multiple negative eigenvalues exist (i.e., in the presence of bifurcation points). In that case,the code will oscillate about this bifurcation point.

For material non linear problems, Bergan [3] suggested that a change in the sign should occur upon reversal of the sign of the incremental work. According to [9], if all increments would be infinitely small and if no bifurcation points are present on

*Key words and phrases.* arc-length method, snap-back, snap-through, bifurcation, non-uniqueness, geometrically and physically non-linear analysis, path following technique, automated solution control.

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the loading path, the incremental work criterion coincides with the appearance of a negative determinant of the tangent stiffness matrix. In [7] however, it is mentioned that the external work sign is deficient in the vicinity of displacement limit points, while the determinant criterion is not. On the basis of the two previous criteria, the following proposal to determine the proper sign of the load estimator is proposed in [2].

$$\lambda_i^1 = +\left|\lambda_i^1\right| \quad if \quad \{\Delta\delta_a\}_{i-1}^{conv}\{\delta_I\}_i^1 \geq 0 \tag{2.1}$$

$$\lambda_i^1 = -\left|\lambda_i^1\right| \quad if \quad \{\Delta\delta_a\}_{i-1}^{conv}\{\delta_I\}_i^1 < 0 \tag{2.2}$$

$\{\Delta\delta_a\}_{i-1}^{conv}$ is the accumulated converged incremental displacement vector of the previous step, $\{\delta_I\}_i^1$ the 'tangent' displacements for this load step and $\lambda_i^1$ the load increment factor (for more details see [10]). This criterion is introduced into LAGAMINE.

The effectiveness of the criterion was confirmed in [17]. However, criteria based on the sign of the incremental work seem to be insensitive (do not respond) to bifurcation according to [8]...

The Euclidean norm of the increment in displacements and forces is used in standard arc-length techniques. As mentioned in [15] and [16], this is not always adequate as it provides a measure of the increment of the solution which is too "global" in contrast with the localized nature of the failure mechanisms. One has to be able to choose another more "local" measure, as the maximum strain increment or the displacements of some specific nodes. The latter is also implemented in LAGAMINE.

## 3. Validation: Brazilian test

The Brazilian test is often used to determine the tensile strength of concrete. As described in [15] and [16], a cylindrical specimen is loaded along a diametral plane, Figure 1. The ideal expected failure mechanism is the splitting of the specimen into two halves along the loading plane, [14], Figure 2.

The finite element mesh used for the calculation is composed of 1718 second gradient elements, [4], [5], [6]. A classical damage constitutive law is used for concrete [11], [12], able to take into account its asymmetric behavior under tension and compression. The behavior of the steel plate is taken linear elastic. Specific geometrical and material parameters are given in Table 1.

Figure 3 presents the applied vertical load versus the vertical displacement of the bearing strip. structural response. Numerical results presents a severe snap-back response and a damage evolution that starts at the center of the specimen and propagates along the loading plane.

## 4. Conclusions-Perspectives

A convergence criterion suitable for material non linearity problems has been introduced into the finite element code LAGAMINE. Using a local second gradient model, implementation is tested on a Brazilian test on a cylindrical concrete specimen. This first validation test is successful in terms of global but also local results. However, further validation tests are necessary ...

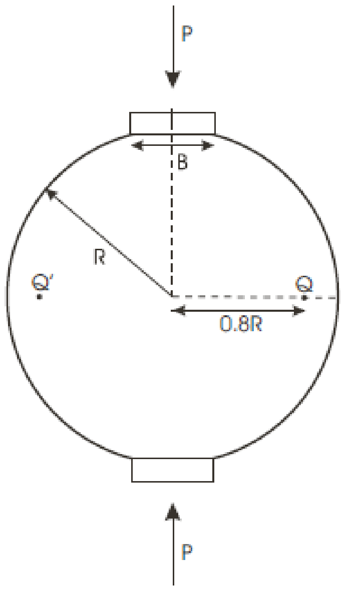


FIGURE 1. *Brazilian test: specimen of radius $R$ and bearing strip of width $B$. After* [15].

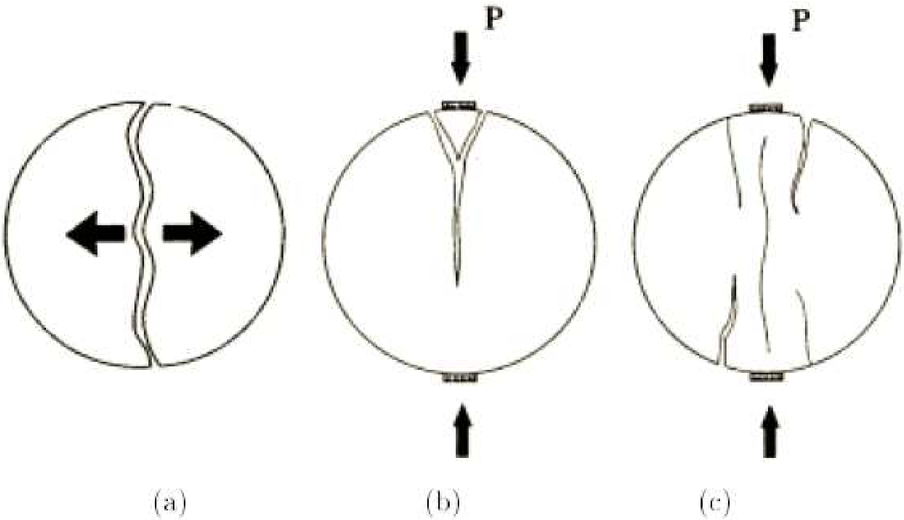


FIGURE 2. *Failure modes in the Brazilian test: (a) ideal single crack splitting along the loading plane; (b) failure mode with wedge formation under the bearing strips; (c) failure mode with primary and secondary crack formations. After* [14].

TABLE 1. Geometry and material parameters.

| *Parameter* | *Symbol* | *Value* |
|---|---|---|
| *Width of bearing strip* | $B$ | $10mm$ |
| *Radius of the specimen* | $R$ | $40mm$ |
| *Young's modulus* | $E$ | $37700\ MPa\ (specimen)$ |
| *Young's modulus* | | $300000\ MPa\ (bearing\ strip)$ |
| *Poisson coefficient* | $\nu$ | $0.2\ (concrete)$ |
| *Poisson coefficient* | $\nu$ | $0.2\ (steel)$ |
| *Damage threshold* | $ed_0$ | $1.e^{-4}$ |
| *Evolution of tensile damage* | $A_t$ | 1 |
| | $B_t$ | 15600 |
| *Evolution of compressive damage* | $A_c$ | 1.4 |
| | $B_c$ | 1900 |
| *Second gradient slope* | $B_{sg}$ | $5000\ N$ |

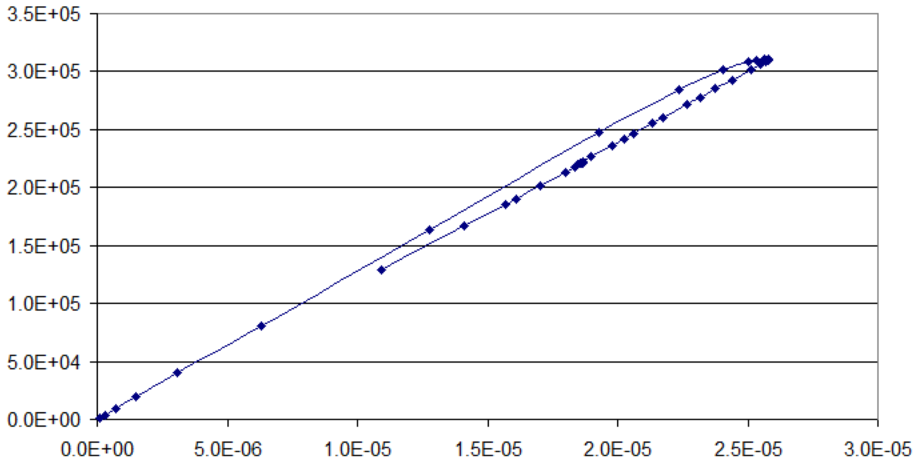


FIGURE 3. *Brazilian test: Vertical load versus vertical displacement.*

Recently, and within the French ANR program MEFISTO ("Maîtrise durable de la fissuration des infrastructures en béton") [13], three point bending tests on concrete beams of different sizes have been performed in Ecole Centrale de Nantes [1]. The evolutions of damage localization patterns and cracks were monitored using digital image correlation techniques. In the near future we will try to reproduce this experimental campaign with LAGAMINE.

## LIST OF FIGURES

## List of Tables



## References

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